

Optimal Power Allocation for GSVD-Based Beamforming in the MIMO Wiretap Channel

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Abstract

This paper considers a multiple-input multiple-output (MIMO) Gaussian wiretap channel model, where there exists a transmitter, a legitimate receiver and an eavesdropper, each equipped with multiple antennas. Perfect secrecy is achieved when the transmitter and the legitimate receiver can communicate at some positive rate, while ensuring that the eavesdropper gets zero bits of information. In this paper, the perfect secrecy capacity of the multiple antenna MIMO wiretap channel is found for arbitrary numbers of antennas under the assumption that the transmitter performs beamforming based on the generalized singular value decomposition (GSVD). More precisely, the optimal allocation of power for the GSVD-based precoder that achieves the secrecy capacity is derived. This solution is shown to have several advantages over prior work that considered secrecy capacity for the general MIMO Gaussian wiretap channel under a high SNR assumption. Numerical results are presented to illustrate the proposed theoretical findings.

Index Terms

MIMO Wiretap Channel, Secrecy Capacity, Physical Layer Security, Generalized Singular Value Decomposition

I. INTRODUCTION

The broadcast nature of a wireless medium makes it very susceptible to eavesdropping, where the transmitted message is decoded by unintended receiver(s). Recent information-theoretic research on secure communication has focused on enhancing security at the physical layer. The wiretap channel, first introduced and studied by Wyner [1], is the most basic physical layer model that captures the problem of communication security. Wyner showed that when an eavesdropper's channel is a degraded version of the main channel, the source and destination can achieve a positive secrecy rate, while ensuring that the eavesdropper gets zero bits of information. The maximum secrecy rate from the source to the destination is defined as the secrecy capacity. The Gaussian wiretap channel, in which the outputs at the legitimate receiver and at the eavesdropper are corrupted by additive white Gaussian noise (AWGN), was studied in [2].

Determining the secrecy capacity of a Gaussian wiretap channel is in general a difficult non-convex optimization problem, and has been addressed independently in [3-7]. Oggier and Hassibi [3] and Khisti and Wornell [4, 5] followed an indirect approach using a Sato-like argument and matrix analysis tools. They considered the problem of finding the secrecy capacity of the Gaussian MIMO wiretap channel subject to a constraint on the total average power, and a closed-form expression for the secrecy capacity in the high signal-to-noise-ratio (SNR) regime was obtained in [5]. However, the optimal input covariance matrix that achieves the secrecy capacity at high SNR is not fully characterized, especially for the case where there is non-trivial nullspace for the channel between the transmitter and eavesdropper. When there is such a nullspace, [5] uses a complicated beamforming matrix to transmit two groups of information-bearing symbols into two different subspaces, one that lies in the nullspace of the channel matrix at the unintended receiver, and the other orthogonal to it. As indicated in [5], most of the transmission power is allocated to the first subspace, and only a small fraction of the power is allocated to the other. Furthermore, the available power is distributed uniformly over the dimensions of the two subspaces. In addition to the

complexity of the beamforming matrix, the other drawback of [5] is that the precise allocation of power between the two subspaces is not clear, nor is the sensitivity of the secrecy capacity to this power fraction quantified.

In [6], Liu and Shamai propose a more information-theoretic approach using the enhancement concept, originally presented by Weingarten et al. [8], as a tool for the characterization of the MIMO Gaussian broadcast channel capacity. Liu and Shamai have shown that an enhanced degraded version of the channel attains the same secrecy capacity as does a Gaussian input distribution. From the mathematical solution in [6] it was evident that such an enhanced channel exists; however it was not clear how to construct such a channel until the work of [7], which provided a closed-form expression for the secrecy capacity under a *covariance matrix* power constraint. While this result is interesting since the expression for the secrecy capacity is valid for all SNR scenarios, there still exists no computable secrecy capacity expression for the MIMO Gaussian wiretap channel under an average total power constraint. To date, a solution has only been obtained for the so-called MISOME case in [4], where the transmitter and eavesdropper may have multiple antennas, but the desired receiver has only one.

In this paper, we investigate the non-convex optimization of the secrecy rate for the general Gaussian multiple-input multiple-output (MIMO) wiretap channel under a constraint on the total average power, where the number of antennas is arbitrary for both the transmitter and the two receivers. We focus on the case where the transmitter uses beamforming (linear precoding) based on the generalized singular value decomposition (GSVD), as in [5]. In particular, we obtain the optimal power allocation that achieves the secrecy capacity for the GSVD scheme. The resulting power allocation is significantly different in nature than the standard water-filling solution for achieving capacity in MIMO links without secrecy considerations. Compared with [5], our beamforming matrix is much simpler to compute, and more importantly, the input covariance matrix that achieves the secrecy capacity is completely characterized in terms of the beamforming and power allocation matrices. We note that the analysis in this paper characterizes the secrecy capacity of the Gaussian MIMO wiretap channel with GSVD-based beamforming for *any* SNR conditions, while [5] gives the secrecy capacity of a *general* Gaussian MIMO wiretap channel (no restriction to GSVD beamforming), but only for the *high* SNR case.

The rest of this paper is organized as follows. In the next section, we describe the assumed mathematical model. We then present the GSVD method and derive the optimal power allocation that achieves the secrecy capacity in Section III. Finally, we demonstrate our results by means of several numerical examples in Section IV.

II. SYSTEM MODEL

Consider a multiple-antenna wiretap channel with n_t transmit antennas and n_r and n_e receive antennas at the legitimate recipient and the eavesdropper, respectively:

$$\mathbf{y}_r = \mathbf{H}_r \mathbf{x} + \mathbf{z}_r \quad (1)$$

$$\mathbf{y}_e = \mathbf{H}_e \mathbf{x} + \mathbf{z}_e \quad (2)$$

where \mathbf{x} is a zero-mean $n_t \times 1$ transmitted signal vector, $\mathbf{z}_r \in \mathbb{C}^{n_r \times 1}$ and $\mathbf{z}_e \in \mathbb{C}^{n_e \times 1}$ are the additive white Gaussian noise (AWGN) vectors at the receiver and eavesdropper, respectively, with i.i.d. entries distributed as $\mathcal{CN}(0, 1)$. The matrices $\mathbf{H}_r \in \mathbb{C}^{n_r \times n_t}$ and $\mathbf{H}_e \in \mathbb{C}^{n_e \times n_t}$ represent the channels associated with the receiver and the eavesdropper, respectively, and are assumed to be quasi-static flat Rayleigh fading and independent of each other, with i.i.d. entries distributed as $\mathcal{CN}(0, \sigma_r^2)$ and $\mathcal{CN}(0, \sigma_e^2)$. Similar to other papers considering the secrecy capacity of the wiretap channel, we assume that the transmitter and both receivers are aware of the channel state information (CSI) for both links.

In a wiretap channel, the transmitter intends to send a confidential message W to the intended receiver while keeping it as secret as possible from the eavesdropper. The corresponding information-theoretic secrecy constraint is given by [1, 9]:

$$\lim_{N \rightarrow \infty} \frac{1}{N} I(W; Y_e^N) = 0 \quad (3)$$

where N is the number of channel uses for sending the message W , and $I(W; Y_e^N)$ represents the mutual information between W and Y_e^N . Consequently, the secrecy capacity is defined as the maximum number of bits that can be correctly transmitted to the intended receiver while keeping the eavesdropper uninformed. Using the single-letter characterization of the secrecy capacity of the wiretap channel provided by Csiszar and Korner in [9], the secrecy capacity is the solution of the following maximization problem:

$$C_{sec} = \max_{U, X} [I(U; Y_r) - I(U; Y_e)] \quad (4)$$

where X , Y_r and Y_e are random variable counterparts to the specific realizations \mathbf{x} , \mathbf{y}_r and \mathbf{y}_e , respectively. U is an auxiliary variable, and the maximization is over all jointly distributed (U, X) such that $U \rightarrow X \rightarrow (Y_r, Y_e)$ forms a Markov chain, while the channel input \mathbf{x} satisfies an average total power constraint

$$\text{Tr}(E\{\mathbf{x}\mathbf{x}^H\}) \leq p \quad (5)$$

where $(.)^H$ denotes the Hermitian (i.e., conjugate) transpose, E is the expectation operator, and $\text{Tr}(\cdot)$ is the matrix trace.

The auxiliary variable U represents a precoding signal. In [6], Liu and Shamai studied the optimization problem of (4) and showed that a Gaussian $U = X$ is an optimal choice. In other words, Gaussian random binning without prefix coding is an optimal coding strategy for the MIMO Gaussian wiretap channel [10]. Hence, a matrix characterization of the secrecy capacity is given by

$$C_{sec} = \max_{\mathbf{Q}_x \succeq 0} [I(X; Y_r) - I(X; Y_e)] \quad (6)$$

where $\mathbf{Q}_x = E\{\mathbf{x}\mathbf{x}^H\}$ is the input covariance matrix. The non-convex maximization problem in (6) is considered under the power constraint (5).

III. OPTIMAL POWER ALLOCATION FOR THE GSVD-BASED GAUSSIAN MIMO WIRETAP CHANNEL

We consider the non-convex maximization problem in (6) for the case that the transmitter performs beamforming by applying the generalized singular value decomposition (GSVD) to the channel matrices \mathbf{H}_r and \mathbf{H}_e . Application of the GSVD technique was first used by Khisti and Wornell (see e.g. [5]) who obtained a closed-form expression for the secrecy capacity in the high SNR regime. In this section, we first describe the GSVD beamforming method and next we derive the optimal power allocation matrix that achieves the secrecy capacity for any SNR, and we describe some important advantages of this scheme over what is proposed in [5].

Definition 1 (GSVD Transform): Given two matrices $\mathbf{H}_r \in \mathbb{C}^{n_r \times n_t}$ and $\mathbf{H}_e \in \mathbb{C}^{n_e \times n_t}$, $gsvd(\mathbf{H}_r, \mathbf{H}_e)$ returns unitary matrices $\Psi_r \in \mathbb{C}^{n_r \times n_r}$ and $\Psi_e \in \mathbb{C}^{n_e \times n_e}$, non-negative diagonal matrices \mathbf{C} and \mathbf{D} , and a matrix $\mathbf{A} \in \mathbb{C}^{n_t \times q}$ with $q = \min(n_t, n_e + n_r)$, such that

$$\mathbf{H}_r \mathbf{A} = \Psi_r \mathbf{C} \quad (7)$$

$$\mathbf{H}_e \mathbf{A} = \Psi_e \mathbf{D} \quad (8)$$

The nonzero elements of \mathbf{C} are in ascending order while the nonzero elements of \mathbf{D} are in descending order. Moreover, $\mathbf{C}^T \mathbf{C} + \mathbf{D}^T \mathbf{D} = \mathbf{I}$.

The transmitted signal vector \mathbf{x} is constructed as

$$\mathbf{x} = \mathbf{A} X_S, \quad X_S \sim \mathcal{CN}(\mathbf{0}, \mathbf{P}) \quad (9)$$

where \mathbf{A} is obtained from $gsvd(\mathbf{H}_r, \mathbf{H}_e)$ as above, and each element of the vector X_S represents an independently encoded Gaussian codebook symbol that is beamformed with the corresponding column of the \mathbf{A} matrix. \mathbf{P} is a positive semi-definite diagonal matrix representing the power allocated by the transmitter to the data symbols. In the following, we derive an optimal source power allocation which

achieves the secrecy capacity of the GSVD-based MIMO Gaussian wiretap channel. Substituting (9) into the channel model (1)-(2) and using (7)-(8) yields

$$\mathbf{y}_r = \mathbf{\Psi}_r \mathbf{C} \mathbf{X}_S + \mathbf{z}_r \quad (10)$$

$$\mathbf{y}_e = \mathbf{\Psi}_e \mathbf{D} \mathbf{X}_S + \mathbf{z}_e \quad (11)$$

Considering the above equations, the maximization problem in (6) is represented by

$$\begin{aligned} C_{sec} &= \max_{\mathbf{Q}_x} [I(X; Y_r) - I(X; Y_e)] = \\ & \max_{\mathbf{P} \succeq \mathbf{0}, \text{diagonal}} \log |\mathbf{I} + \mathbf{\Psi}_r \mathbf{C} \mathbf{P} \mathbf{C}^T \mathbf{\Psi}_r^H| - \log |\mathbf{I} + \mathbf{\Psi}_e \mathbf{D} \mathbf{P} \mathbf{D}^T \mathbf{\Psi}_e^H| \\ & \text{subject to } \text{Tr}(\mathbf{A}^H \mathbf{A} \mathbf{P}) \leq p \end{aligned} \quad (12)$$

Theorem 1: Assuming that the transmitter applies the proposed beamforming matrix \mathbf{A} , the optimal source power allocation \mathbf{P}^* that achieves the secrecy capacity in problem (12) is given by

$$p_i^* = \begin{cases} \max(0, \frac{-1 + \sqrt{1 - 4c_i d_i + 4(c_i - d_i)c_i d_i / (\mu a_i)}}{2c_i d_i}), & \text{if } c_i > d_i \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

where p_i^* , c_i , d_i and a_i are the i th diagonal elements of the matrices \mathbf{P}^* , $\mathbf{C}^T \mathbf{C}$, $\mathbf{D}^T \mathbf{D}$ and $\text{diag}(\mathbf{A}^H \mathbf{A})$, respectively. The Lagrange parameter $\mu > 0$ is chosen to satisfy the power constraint (5).

Proof: The optimization problem is non-convex. Our proof technique involves applying the Karush-Kuhn-Tucker (KKT) conditions (as necessary conditions), which help express the Lagrangian in the form of an integral. This specific structure of the problem is then exploited to obtain a closed-form solution for the optimal power allocation strategy. Details can be found in the Appendix.

Eqs. (7) and (8) show that applying the GSVD transform to \mathbf{H}_r and \mathbf{H}_e simultaneously diagonalizes them. Thus, the GSVD transform creates a set of parallel independent subchannels between the sender and the receivers, and it suffices to use independent Gaussian codebooks across these subchannels. More precisely, as (13) indicates, it is optimal to use only those subchannels for which the output at the eavesdropper is a degraded version of the output at the destination node. These subchannels correspond to the condition $c_i > d_i$, or as shown in [5], generalized singular values of $gsvd(\mathbf{H}_r, \mathbf{H}_e)$ which are larger than 1. Clearly, if there are no such subchannels, the achievable secrecy rate using this transmission scheme would be zero [11].

It is interesting to note that the optimal source power allocation (13) is different from the water-filling allocation that achieves capacity for fading channels without the secrecy constraint. Moreover, as we will observe in Section IV, (13) has an important role in achieving the secrecy capacity even for moderately high SNRs. We have the following result.

Corollary 1: The secrecy capacity of the GSVD-based Gaussian MIMO wiretap channel is

$$C_{sec} = \log |\mathbf{I} + \mathbf{P}^* \mathbf{C}^T \mathbf{C}| - \log |\mathbf{I} + \mathbf{P}^* \mathbf{D}^T \mathbf{D}| \quad (14)$$

Proof: Follows directly from substituting (13) in to (12) and by considering the fact that $\mathbf{\Psi}_r$ and $\mathbf{\Psi}_e$ are unitary matrices.

It was shown in [5] that GSVD beamforming with uniform power allocation is sufficient to attain the secrecy capacity of a MIMO Gaussian wiretap channel in the high SNR regime. However, for the case where there is a non-trivial nullspace in the channel between the transmitter and eavesdropper, the optimal input covariance matrix that achieves the secrecy capacity is not fully characterized in [5]. When there is such a nullspace, [5] uses a complicated beamforming matrix to transmit two groups of information bearing symbols into two different subspaces. The associated beamforming matrices are obtained by

performing an LQ decomposition on \mathbf{A} as well as additional calculations which lead to Eq. (59) in [5]. The aforementioned subspaces are identified as follows [5, Eq. (58)]:

$$\begin{aligned}\mathcal{S}_1 &= \text{Null}(\mathbf{H}_e) \cap \text{Null}(\mathbf{H}_r)^\perp \\ \mathcal{S}_2 &= \text{Null}(\mathbf{H}_e)^\perp \cap \text{Null}(\mathbf{H}_r)^\perp\end{aligned}\tag{15}$$

where $\text{Null}(\cdot)$ denotes the nullspace of its matrix argument, while \perp denotes the orthogonal complement. It is important to note that our transmission scheme in (9) and consequently our solution in Theorem 1 does not require such calculations, and in fact yields the secrecy capacity of the Gaussian MIMO wiretap channel with GSVD-based beamforming for any SNR under any assumptions regarding the nullspace of the transmitter-to-eavesdropper channel.

As indicated in [5], most of the transmission power is assumed to be allocated to the subspace \mathcal{S}_1 and only a small fraction for \mathcal{S}_2 , and the available power is distributed uniformly over the dimensions of each of these two subspaces. The exact allocation of power between these two subspaces is not specified in [5], nor is the sensitivity of the secrecy capacity to the power allocation studied. In the next section, we show that the secrecy capacity of a MIMO Gaussian wiretap channel is in fact quite sensitive to how power is allocated between these two subspaces, which illustrates that using the proposed optimal power allocation is essential.

IV. NUMERICAL RESULTS

In this section, we present numerical results to illustrate our theoretical findings. In all of the following figures, the channel matrices and background noise are modeled in the same way that we described in Section II. In each figure, the values of n_t , n_r and n_e , as well as σ_r^2 , σ_e^2 and p , will be depicted. In the simulations we compare the secrecy capacity obtained by the optimal power allocation with the secrecy rate achieved by uniform power allocation for a MIMO Gaussian wiretap channel with GSVD-based beamforming. This comparison is performed for various transmit powers (SNR), channel conditions and also different numbers of antennas for the transmitter and receivers. All displayed results are calculated based on an average of at least 100 independent channel realizations. In each trial, the secrecy capacity is obtained by evaluating (14), while the secrecy rate achieved by the uniform power allocation is obtained by using analytical results presented in [5].

First we consider the case where the nullspace of the eavesdropper's channel is non-trivial. Figs. 1 and 2 investigate the allocation of power between subspaces \mathcal{S}_1 and \mathcal{S}_2 as defined in (15), for a case where $n_t = n_r = 5$, $n_e = 4$ and the transmit power is $p = 100$, or SNR=20 dB. The solid horizontal line illustrates the secrecy capacity of a Gaussian MIMO wiretap channel with GSVD-based beamforming and optimal power allocation, while the dashed curve represents the secrecy rate achieved by uniform power allocation versus the fraction of power used in subspace \mathcal{S}_2 . Note that, for this high SNR scenario, the peak of this curve is the secrecy capacity of the *general* MIMO Gaussian wiretap channel [5]. Note also that, contrary to claims made in [5], the secrecy rate is quite sensitive to the fraction of power allocated to the two subspaces, and optimal performance requires a non-trivial allocation of power to \mathcal{S}_2 (over 20%). The advantage of the optimal GSVD-based power allocation approach is that this imprecise distribution of power to the two subspaces is eliminated.

Fig. 3 compares the secrecy capacity achieved by the optimal power allocation and the secrecy rate achieved by the uniform power allocation for different transmit powers (SNRs). In this example, there is no non-trivial null space between the transmitter and eavesdropper. The figure shows that the difference between the optimal and uniform power allocation is important, even at moderately high SNRs. This is especially true for the case where the desired receiver's channel is of equal or better quality than the eavesdropper's channel ($\sigma_r^2 \geq \sigma_e^2$). As the figure shows, the performance difference between the optimal and uniform power allocation curves slowly decreases as the SNR is increased. This is due to this fact that, as derived in [5] for the high SNR regime, uniform power allocation is sufficient to achieve the secrecy capacity.

V. CONCLUSIONS

We have established the secrecy capacity for the Gaussian MIMO wiretap channel assuming the transmitter uses GSVD-based beamforming. This non-convex optimization problem is solved subject to an average transmit power constraint. In particular, we have derived the optimal power allocation for the GSVD-based beamformers that achieves the secrecy capacity. Our numerical results demonstrate that the optimal power allocation is necessary even for relatively high SNRs.

APPENDIX

We are interested in obtaining the power distribution that achieves the secrecy capacity of the Gaussian MIMO wiretap channel in problem (12) for the GSVD-based beamforming scheme presented in Section III. This non-convex optimization problem is to be solved with the average power constraint

$$\begin{aligned}\text{Tr}(E\{\mathbf{x}\mathbf{x}^H\}) &= \text{Tr}(\mathbf{A}E\{X_S X_S^H\}\mathbf{A}^H) \\ &= \text{Tr}(\mathbf{A}\mathbf{P}\mathbf{A}^H) = \text{Tr}(\mathbf{A}^H\mathbf{A}\mathbf{P}) \leq p\end{aligned}\quad (16)$$

By considering the fact that Ψ_r and Ψ_e are unitary matrices and $\det(\mathbf{I} + \mathbf{E}\mathbf{F}) = \det(\mathbf{I} + \mathbf{F}\mathbf{E})$, the Lagrangian function \mathcal{L} associated with this problem is given by

$$\mathcal{L} = \log |\mathbf{I} + \mathbf{P}\mathbf{C}^T\mathbf{C}| - \log |\mathbf{I} + \mathbf{P}\mathbf{D}^T\mathbf{D}| - \mu \text{Tr}(\mathbf{A}^H\mathbf{A}\mathbf{P}) \quad (17)$$

where $\mu > 0$ is the Lagrange multiplier. Since \mathbf{P} , $\mathbf{C}^T\mathbf{C}$ and $\mathbf{D}^T\mathbf{D}$ are diagonal matrices, \mathcal{L} can be written as

$$\mathcal{L} = \sum_i [\log(1 + p_i c_i) - \log(1 + p_i d_i)] - \mu \sum_i a_i p_i \quad (18)$$

where p_i , c_i , d_i and a_i are the i th diagonal elements of the matrices \mathbf{P} , $\mathbf{C}^T\mathbf{C}$, $\mathbf{D}^T\mathbf{D}$ and $\text{diag}(\mathbf{A}^H\mathbf{A})$, respectively. Clearly, p_i , c_i , d_i and a_i all are real non-negative numbers. Using a technique similar to that proposed in [12], the optimal p_i^* must maximize

$$\mathcal{L}_i = \log(1 + p_i c_i) - \log(1 + p_i d_i) - \mu a_i p_i = \int_0^{p_i} f_i(x) dx \quad (19)$$

where $f_i(x)$ is defined as

$$f_i(x) = \frac{1}{\ln 2} \left(\frac{c_i}{1 + x c_i} - \frac{d_i}{1 + x d_i} \right) - \mu a_i \quad (20)$$

To obtain the optimal p_i^* that maximizes \mathcal{L}_i in (19), we consider two cases based on the relationship between c_i and d_i .

- 1) $c_i \leq d_i$: In this case, (20) is always non-positive, i.e., $f_i(x) \leq 0$. Hence, the maximum of \mathcal{L}_i is achieved by $p_i^* = 0$.
- 2) $c_i > d_i$: In this case, since $f_i(x)$ is a decreasing function for $x \geq 0$, the optimal p_i^* that maximizes \mathcal{L}_i depends on the value of the largest root of $f_i(x) = 0$. Let x_{Li} denote the largest root of $f_i(x) = 0$, i.e.,

$$x_{Li} = \frac{-1 + \sqrt{1 - 4c_i d_i + 4(c_i - d_i)c_i d_i / (\mu a_i)}}{2c_i d_i}$$

where we have used the fact that $\mathbf{C}^T\mathbf{C} + \mathbf{D}^T\mathbf{D} = \mathbf{I}$, or equivalently $c_i + d_i = 1$.¹ If x_{Li} is positive then the maximum of \mathcal{L}_i is achieved by $p_i^* = x_{Li}$, otherwise $p_i^* = 0$.

¹It is easy to verify that $\mu > 0$ guarantees $1 - 4c_i d_i + 4(c_i - d_i)c_i d_i / (\mu a_i) \geq 0$ for the case $c_i > d_i$. To do so, it is sufficient to prove that $1 - 4c_i d_i \geq 0$. We have:

$$c_i + d_i = 1 \rightarrow 1 - 4c_i d_i = 1 - 4(1 - d_i)d_i = 1 - 4d_i + 4d_i^2 = (1 - 2d_i)^2 \geq 0$$

Combining cases 1 and 2, we obtain the desired result as

$$p_i^* = \begin{cases} \max(0, \frac{-1 + \sqrt{1 - 4c_i d_i + 4(c_i - d_i)c_i d_i / (\mu a_i)}}{2c_i d_i}), & \text{if } c_i > d_i \\ 0, & \text{otherwise} \end{cases} \quad (21)$$

Finally, the Lagrange parameter $\mu > 0$ is chosen to satisfy the power constraint (16).

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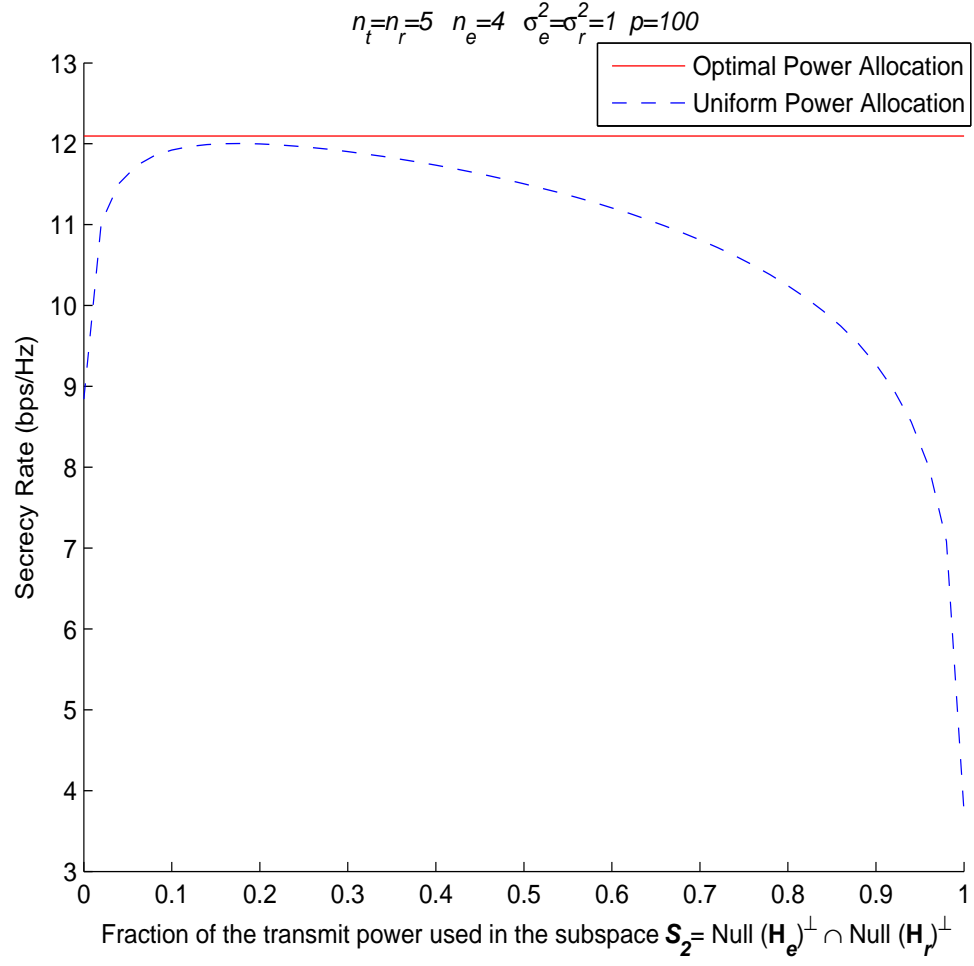


Fig. 1. Comparison of secrecy capacity for optimal power allocation with secrecy rate for uniform power allocation at high SNR in a low interference scenario.

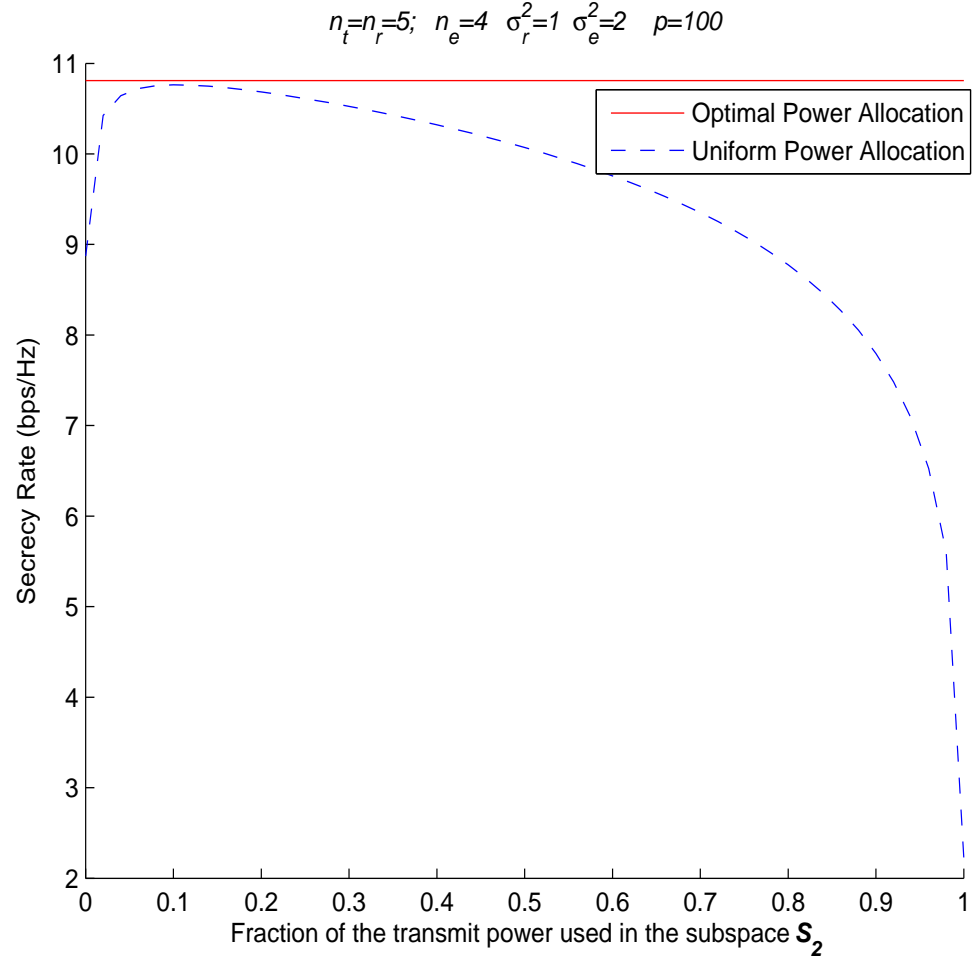


Fig. 2. Comparison of secrecy capacity for optimal power allocation with secrecy rate for uniform power allocation at high SNR in a high interference scenario.

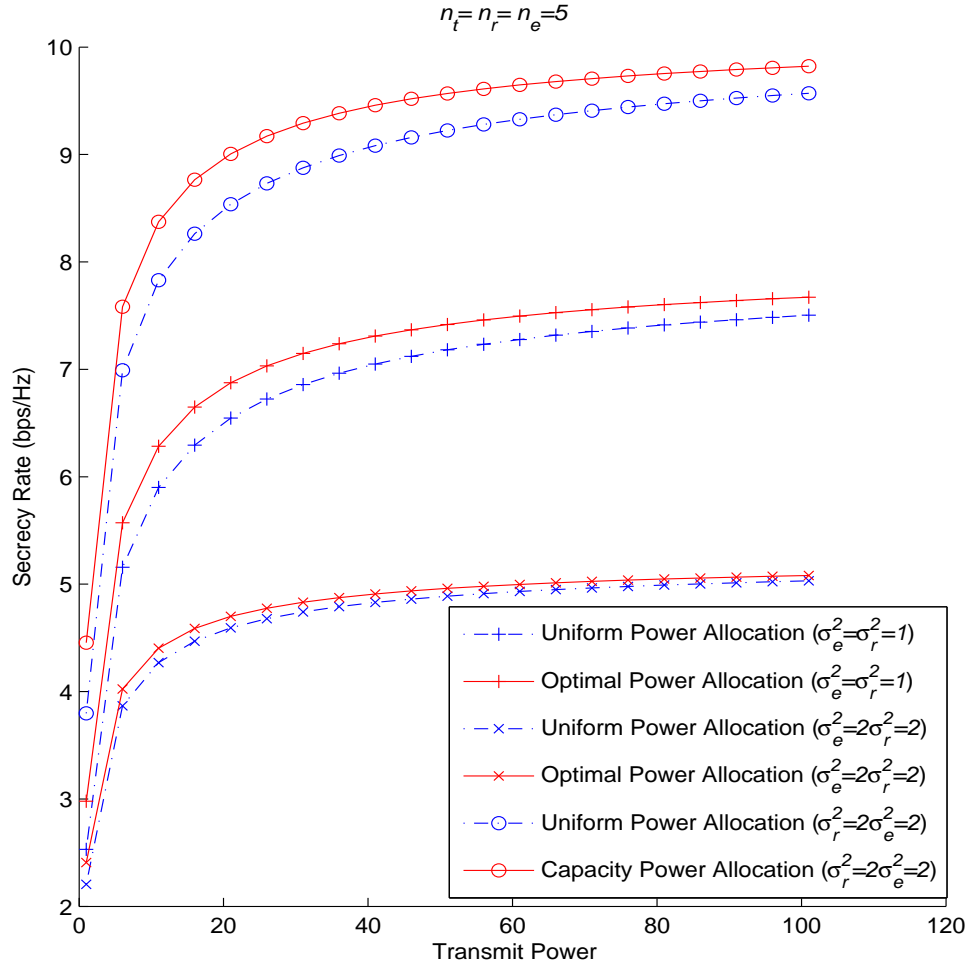


Fig. 3. Comparison of secrecy capacity by optimal power allocation with secrecy rate by uniform power allocation under different SNR and interference situations.